Top PDFs

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Next Steps in the Energy Frontier
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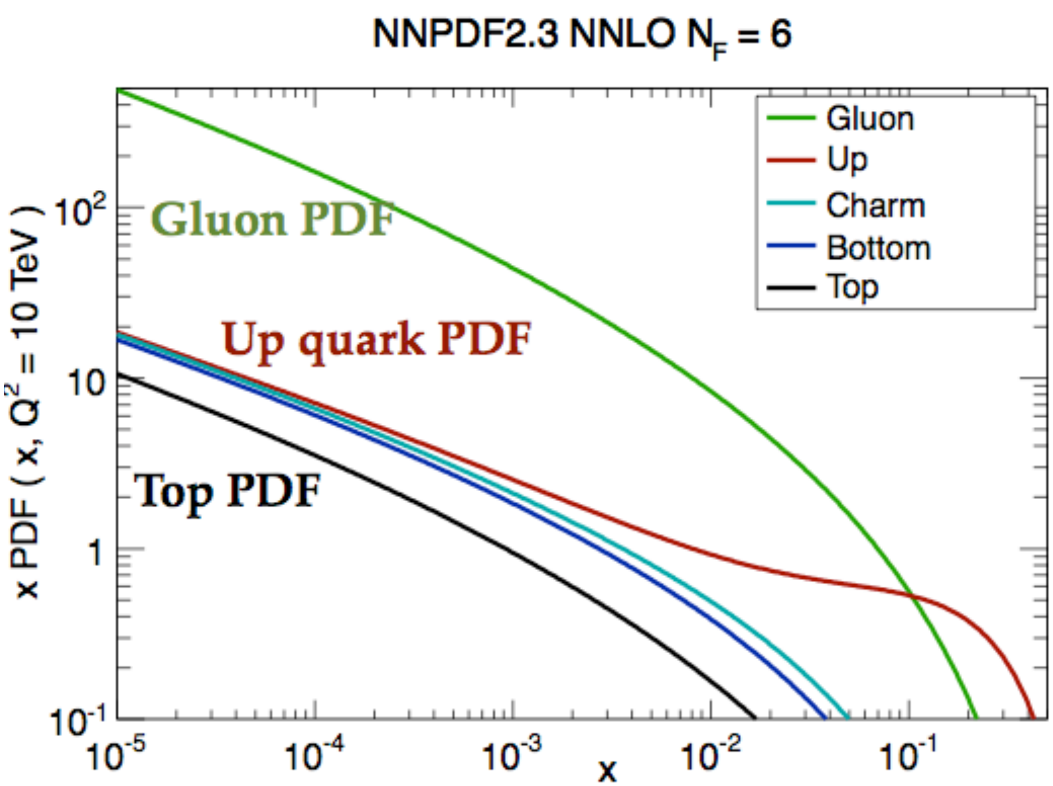
1405.6211
with Sally Dawson and Ian Low

A 100 TeV pp collider

- At 100 TeV, even “heavy” quarks have masses below scales of new processes
- Do we need to consider a top PDF?
- Most PDF sets only include five flavors

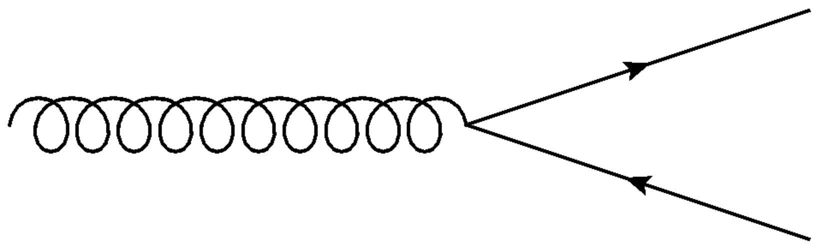


If included, top PDF is non-trivial in size at high scales

J. Rojo, Future Circular Collider Study Kickoff Meeting

Heavy quark PDFs

- Arise from gluon splitting at scales above quark mass



- Should be able to approximate heavy quark PDF

$$\tilde{f}_Q(x,\mu) = \frac{\alpha_s(\mu)}{2\pi} \log \frac{\mu^2}{m_Q^2} \int_x^1 \frac{dz}{z} P_{qg}(z) f_g(\frac{x}{z},\mu)$$

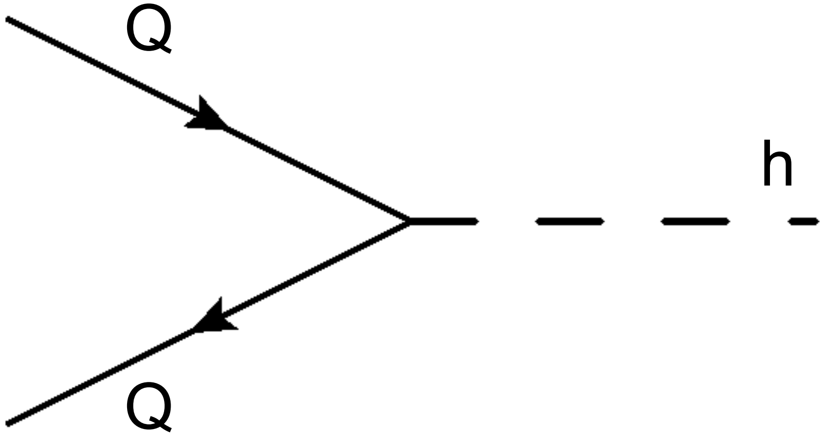
splitting function

$$P_{qg}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

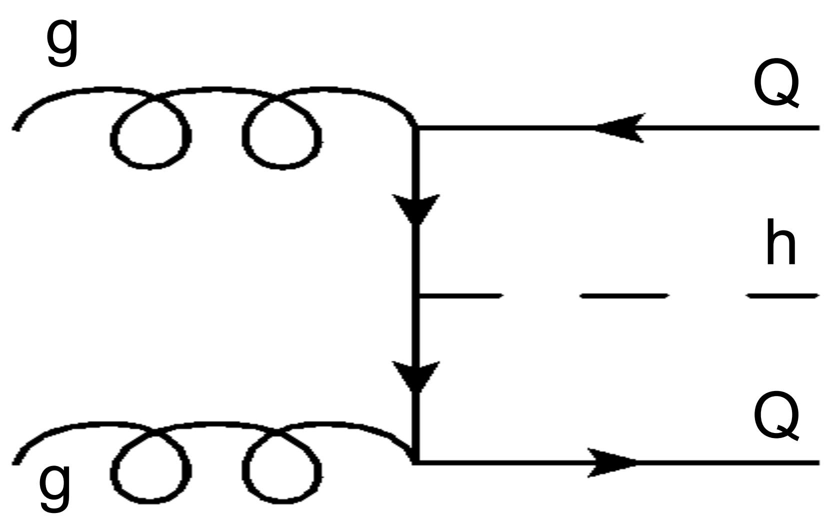
gluon PDF

Heavy quark PDFs

- If we could calculate to infinite order, it wouldn't matter whether we used a heavy quark PDF or not
- As an example, consider h + X production in the PDF schemes with and without the heavy quark



Massless scheme
NF = N

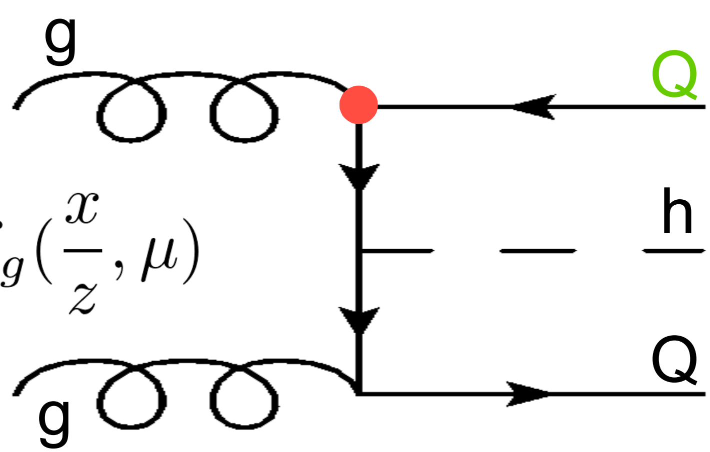


Massive scheme
NF = N - 1

Heavy quark PDFs

- In the scheme without a heavy quark PDF, the leading diagram for h + X production has a collinear divergence
- When we integrate over the phase space for Q, we pick up a factor log(m_h / m_Q), as the quark mass regulates this divergence
- At large m_h, this is just the approximate heavy quark distribution

$$\tilde{f}_Q(x,\mu) = \frac{\alpha_s(\mu)}{2\pi} \log \frac{\mu^2}{m_Q^2} \int_x^1 \frac{dz}{z} P_{qg}(z) f_g(\frac{x}{z},\mu)$$



Heavy quark PDFs

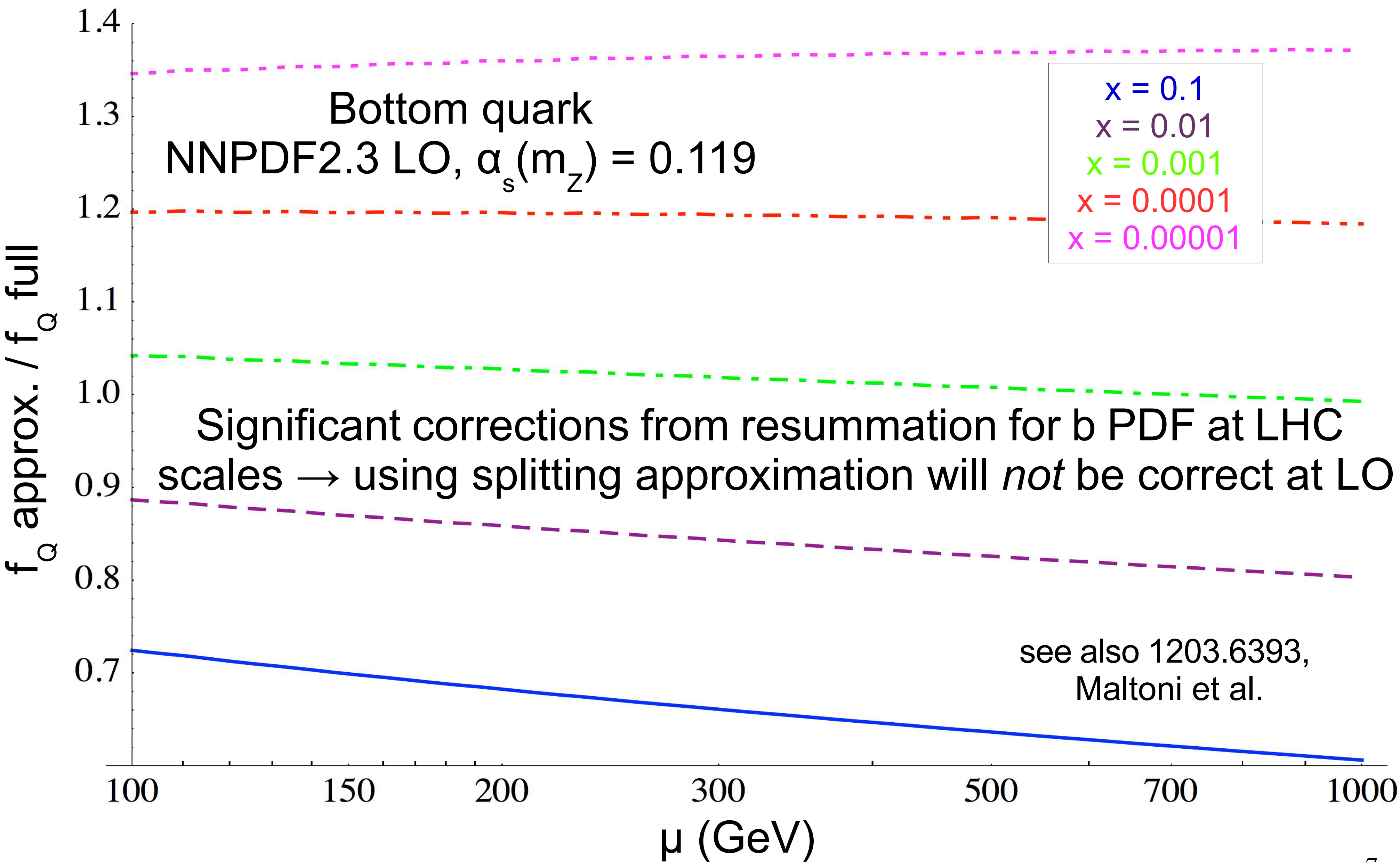
- To get the full heavy quark PDF at leading order, we would have to numerically solve the LO DGLAP equations

$$\frac{d}{d\log\mu^2} f_Q(x,\mu) = \frac{\alpha_s(\mu)}{2\pi}\int_x^1 \frac{dz}{z}\Big(P_{qq}(z) f_Q(\frac{x}{z},\mu) + P_{qg}(z) f_g(\frac{x}{z},\mu)\Big)$$

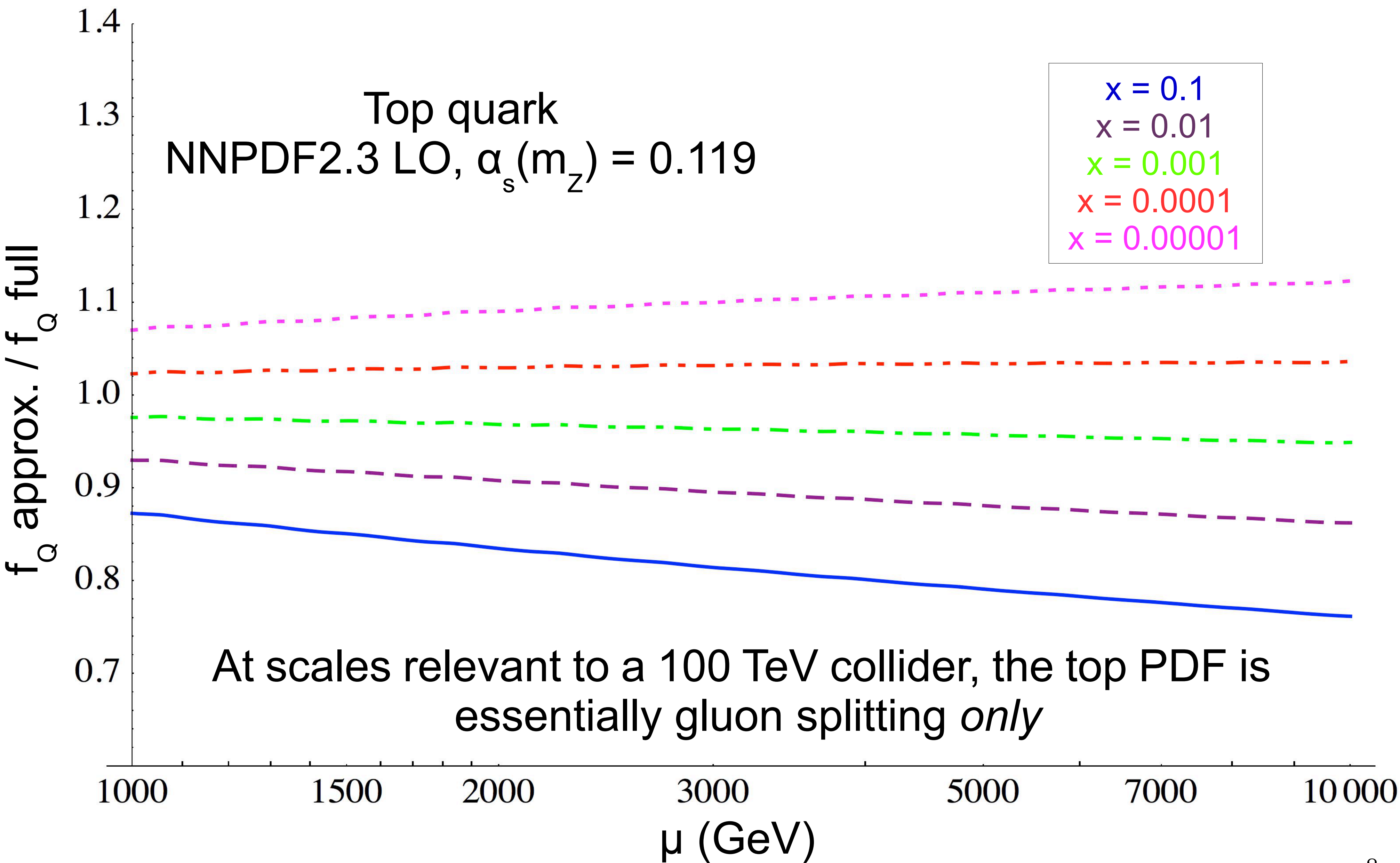
$$f_Q(x, m_Q) = 0$$

- Physically, the difference between our approximation and the full LO heavy quark PDF is the resummation of the logarithms corresponding to multiple parton splittings that are strongly ordered
- How important is this resummation?

Heavy quark PDFs



Heavy quark PDFs



At scales relevant to a 100 TeV collider, the top PDF is essentially gluon splitting *only*

Heavy quark PDFs

- The approximate top PDF at 100 TeV works better than the approximate bottom PDF at the LHC
- The difference can be attributed to the fact that $\alpha_s(\mu) \log(\mu / m_Q)$ is smaller in the former case
- So we should expect that in general, the 5- and 6-flavor schemes give similar results at a 100 TeV collider for processes involving top quarks
- Only at very high scales, when the log gets large, should there be any appreciable difference between the schemes

Charged Higgs production

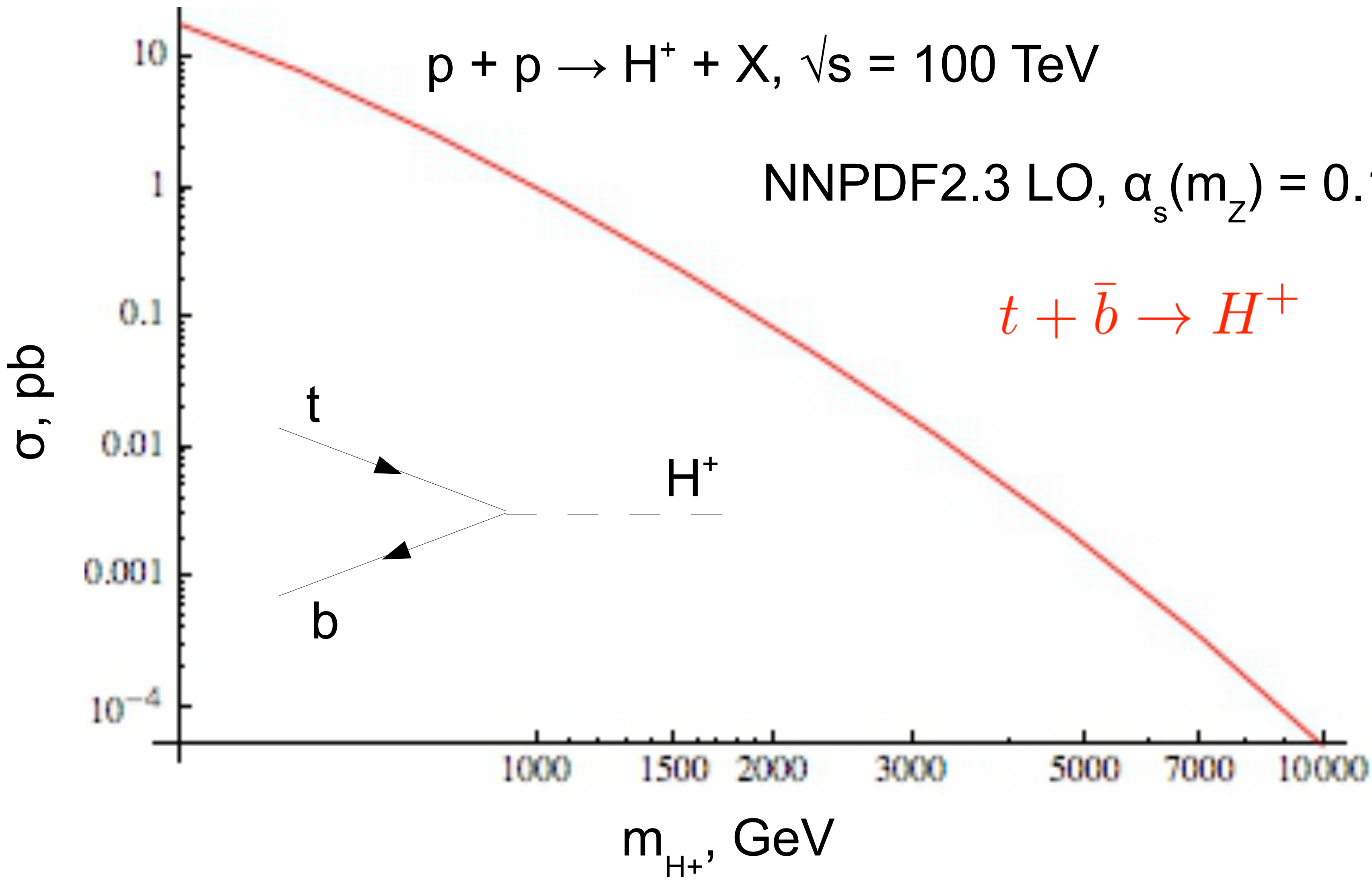
- We can now apply our PDF studies to a sample process at 100 TeV
- Charged Higgses are generic in models with additional Higgs multiplets, with significant couplings to heavy quarks
- To what extent must we calculate H^+ production using a top PDF?

Barnett, Haber and Soper, Nucl. Phys. B306 (1988) 697
Olness and Tung, Nucl. Phys. B308 (1988) 813

- We will outline the computation of the cross section in the NF = 6 scheme, including the top PDF
- Assume MSSM-type couplings with tan β = 5 for numerics, but this is just an overall factor

Charged Higgs production

- Leading diagram is $t + \bar{b} \to H^+$



Charged Higgs production

- Can organize terms in charged Higgs production cross section according to powers of strong coupling and large logs; first term in 6FNS gives leading log
- The different flavor number schemes sum these terms differently, but of course the final results would be identical if we could work to infinite order

Powers of strong coupling $\rightarrow$

Fewer logs $\rightarrow$

$\alpha_s \log \frac{m_H}{m_t}$	$\alpha_s^2 \log^2 \frac{m_H}{m_t}$	$\alpha_s^3 \log^3 \frac{m_H}{m_t}$	$\dots$
α_s	$\alpha_s^2 \log \frac{m_H}{m_t}$	$\alpha_s^3 \log^2 \frac{m_H}{m_t}$	$\dots$
	α_s^2	$\alpha_s^3 \log \frac{m_H}{m_t}$	$\dots$
$t + \bar{b} \rightarrow H^+$		$\dots$	$\dots$

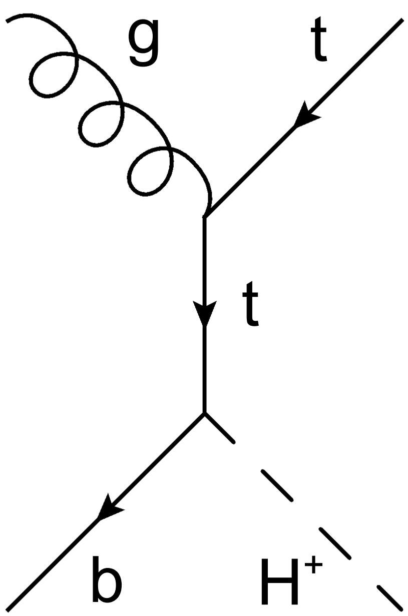
Charged Higgs production

- In 6FNS, next we have $g + \bar{b} \to \bar{t} + H^+$

  (note this is the leading diagram for NF = 5)

- In the limit $m_t \to 0$, this process has a divergence, but it's regulated by the top mass

- Adding it to the previous process would be double-counting the collinear gluon splitting



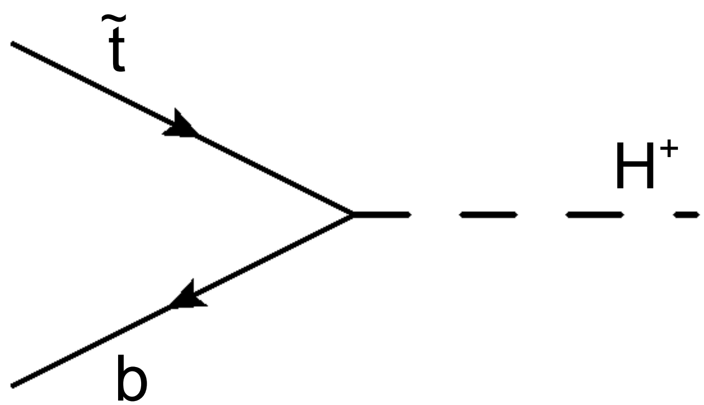
$$\begin{array}{cccc}
\alpha_s \log \frac{m_H}{m_t} & \alpha_s^2 \log^2 \frac{m_H}{m_t} & \alpha_s^3 \log^3 \frac{m_H}{m_t} & \cdots \\
\alpha_s & \alpha_s^2 \log \frac{m_H}{m_t} & \alpha_s^3 \log^2 \frac{m_H}{m_t} & \cdots \\
 & \alpha_s^2 & \alpha_s^3 \log \frac{m_H}{m_t} & \cdots \\
 & & \cdots & \cdots
\end{array}$$

Charged Higgs production

- To avoid double-counting, need to perform subtraction
- Use approximate top PDF

$$\tilde{f}_Q(x,\mu) = \frac{\alpha_s(\mu)}{2\pi}\log\frac{\mu^2}{m_Q^2}\int_x^1 \frac{dz}{z}P_{qg}(z)f_g(\frac{x}{z},\mu)$$

Subtract from sum of previous two processes

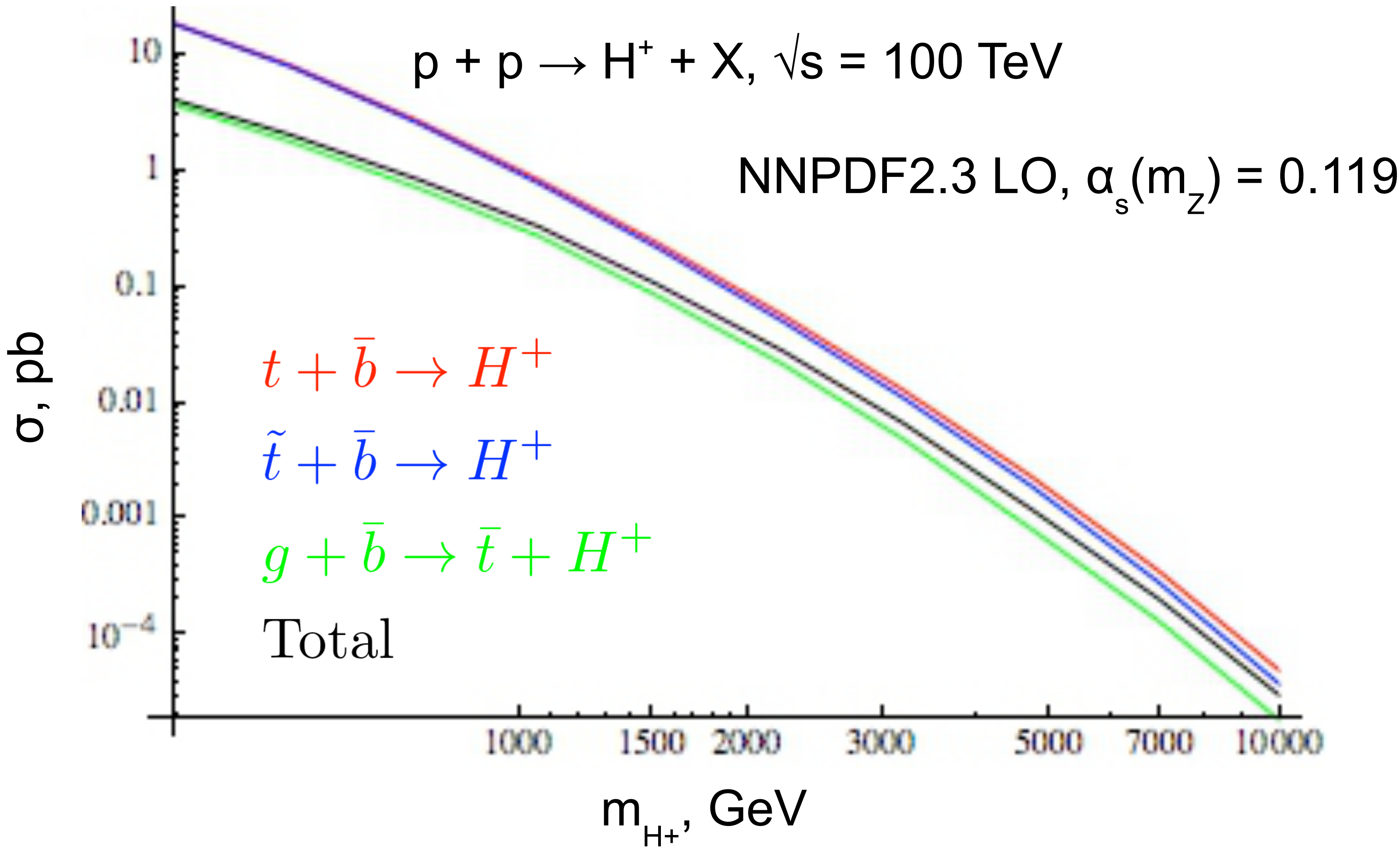


$\alpha_s \log \frac{m_H}{m_t}$	$\alpha_s^2 \log^2 \frac{m_H}{m_t}$	$\alpha_s^3 \log^3 \frac{m_H}{m_t}$	$\ldots$
α_s	$\alpha_s^2 \log \frac{m_H}{m_t}$	$\alpha_s^3 \log^2 \frac{m_H}{m_t}$	$\ldots$
	α_s^2	$\alpha_s^3 \log \frac{m_H}{m_t}$	$\ldots$
		$\ldots$	$\ldots$

$\tilde{t} + \bar{b} \to H^+$

Charged Higgs production

- Subtraction term matches leading log well up to high scales, indicating negligible resummation effects



Charged Higgs production

- As expected, the full top PDF is well approximated by single gluon splitting, and the difference between full LL and gluon splitting is only significant at large scales
- This indicates that the effect of resumming large logs coming from the top phase space is small
- In fact, phase space suppression yields a log even smaller than the ratio of scales we would roughly estimate

$$\log \frac{\mu^2}{m_t^2} \leftarrow \mu^2 = m_{H^+}^2 \frac{(1-z)^2}{z}, z = m_{H^+}^2/\hat{s}$$

- This phase space suppression is generic for processes involving heavy quarks

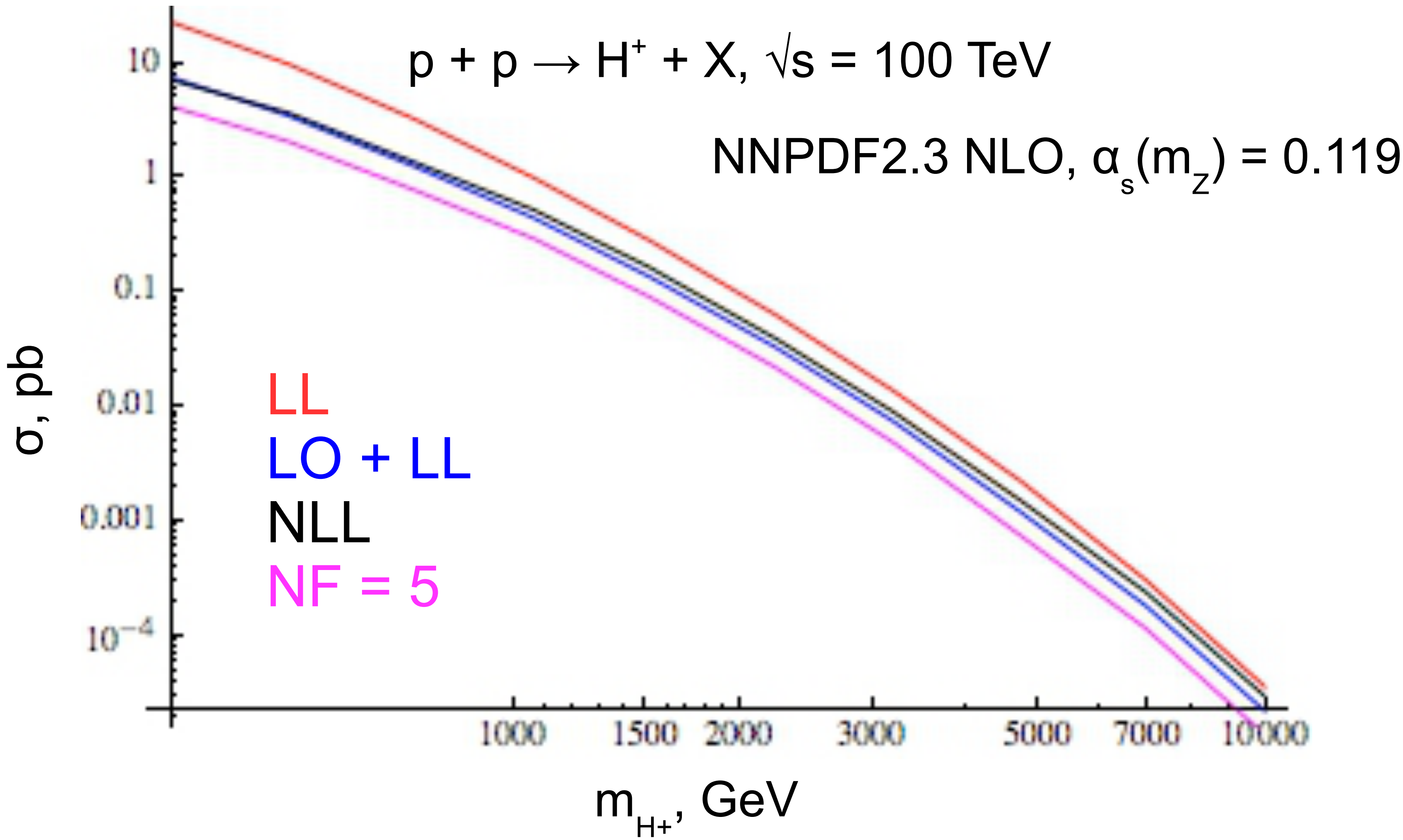
Charged Higgs production

- The cross section is now complete up to terms of order α_s^2 ($\log m_H / m_t$) and higher
- Full NLL requires a few more components
  - NLO PDFs rather than LO PDFs
  - The log-suppressed process $g + t \to b + H^+$ with the appropriate subtraction term
  - The virtual and real corrections to $t + \bar{b} \to H^+$

$\alpha_s \log \frac{m_H}{m_t}$	$\alpha_s^2 \log^2 \frac{m_H}{m_t}$	$\alpha_s^3 \log^3 \frac{m_H}{m_t}$	$\ldots$
α_s	$\alpha_s^2 \log \frac{m_H}{m_t}$	$\alpha_s^3 \log^2 \frac{m_H}{m_t}$	$\ldots$
	α_s^2	$\alpha_s^3 \log \frac{m_H}{m_t}$	$\ldots$
		$\ldots$	$\ldots$

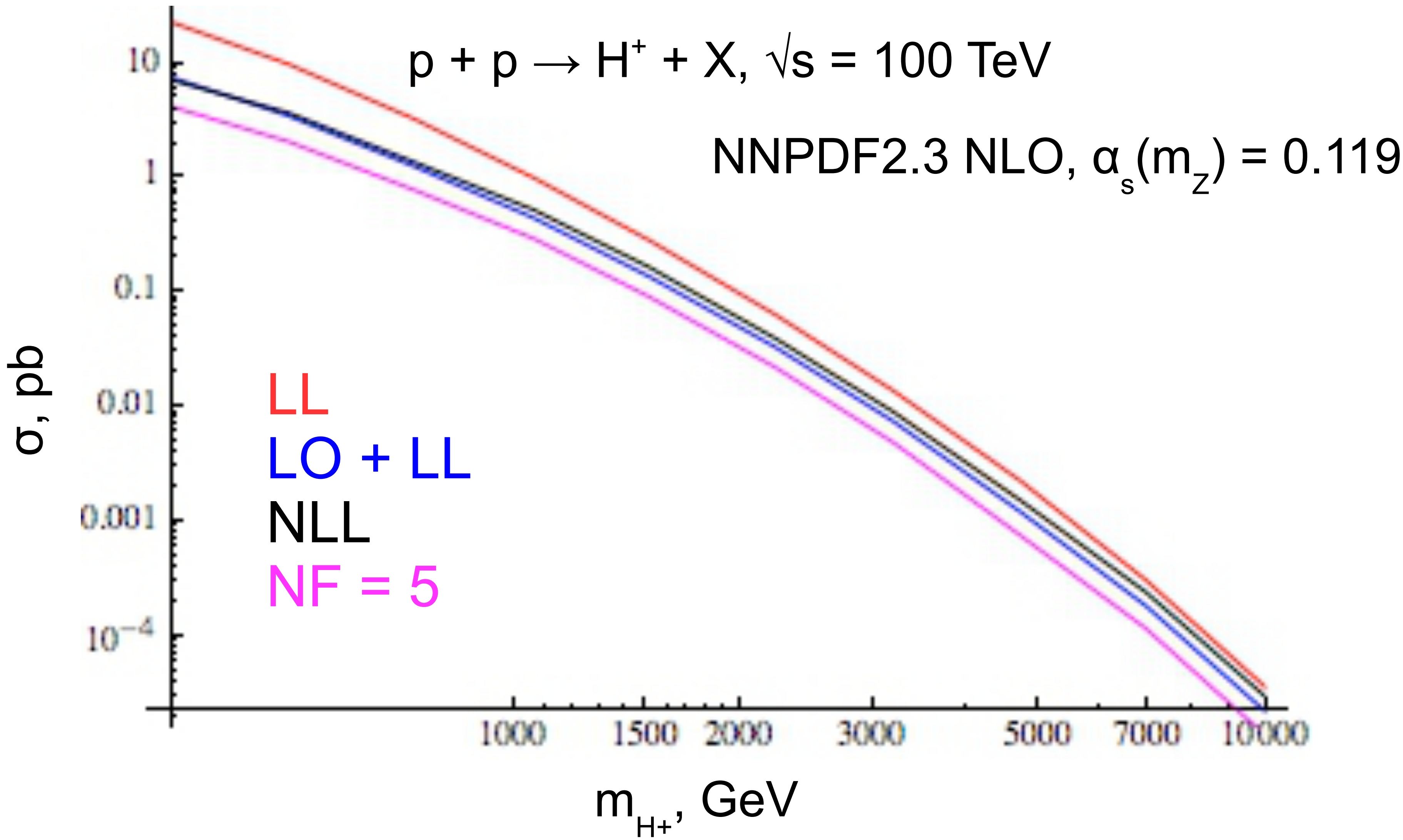
Charged Higgs production

- Going from LO + LL to full NLL doesn't change much, indicating that the perturbation series is under control



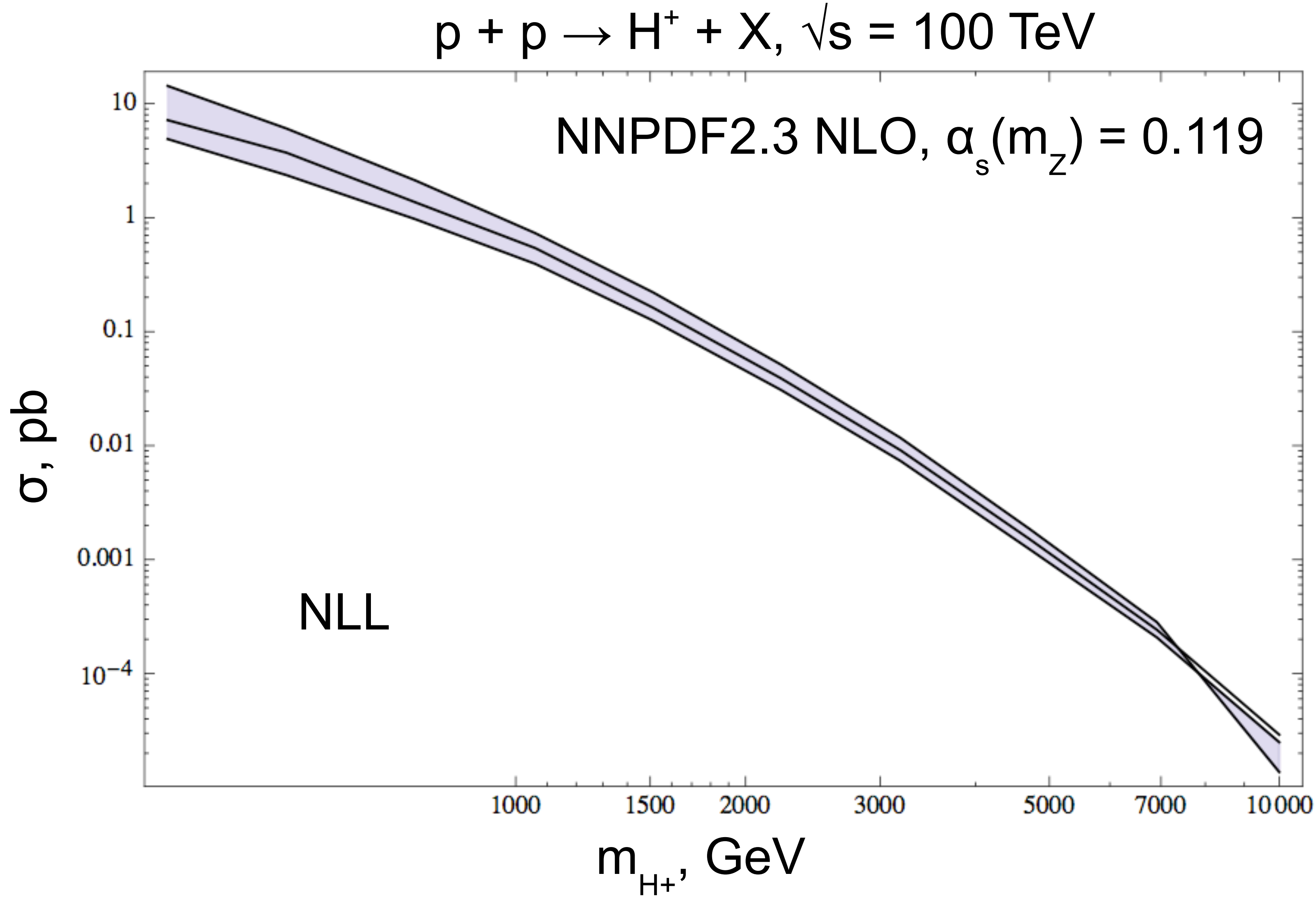
Charged Higgs production

- Total cross section is well approximated by the NF = 5 scheme up to factors of a few at very large H^+ mass



Charged Higgs production

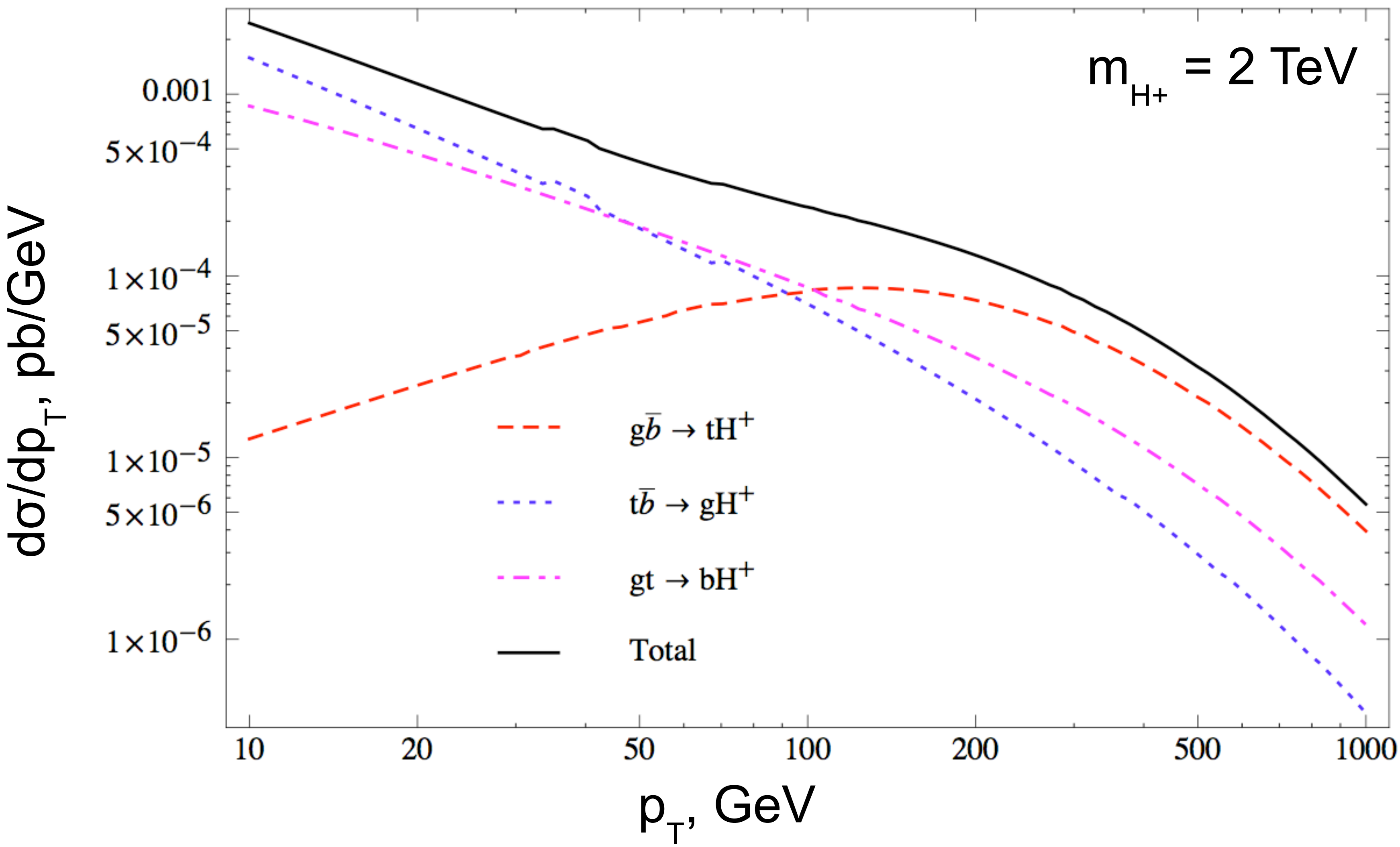
- At high charged Higgs mass, differences between schemes is small compared to scale uncertainty



Charged Higgs production

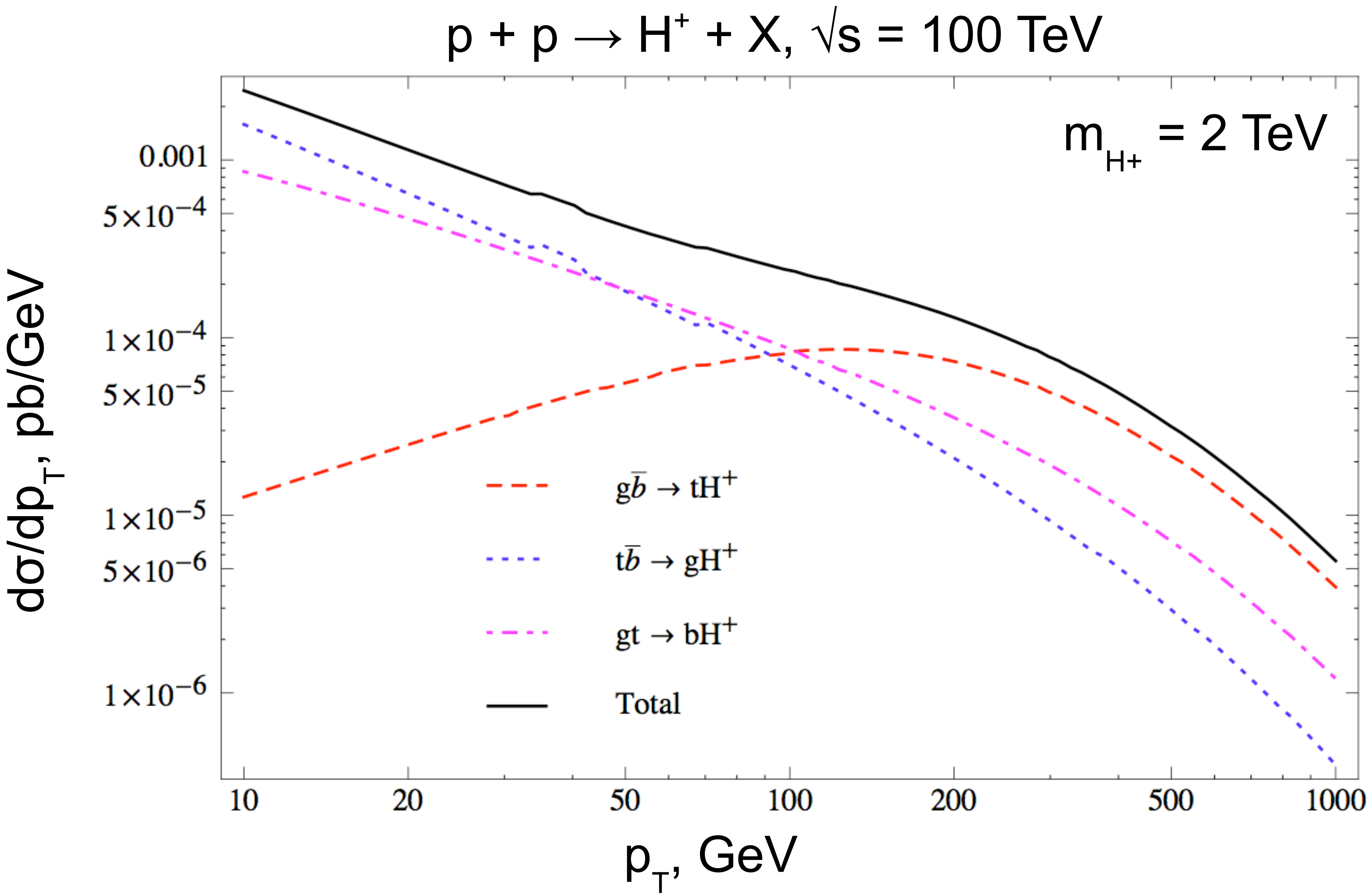
- Higgs p_T spectrum dominated by gluon emission at low p_T, which doesn't exist at LO in NF = 5 scheme

$p + p \rightarrow H^+ + X$, $\sqrt{s} = 100$ TeV



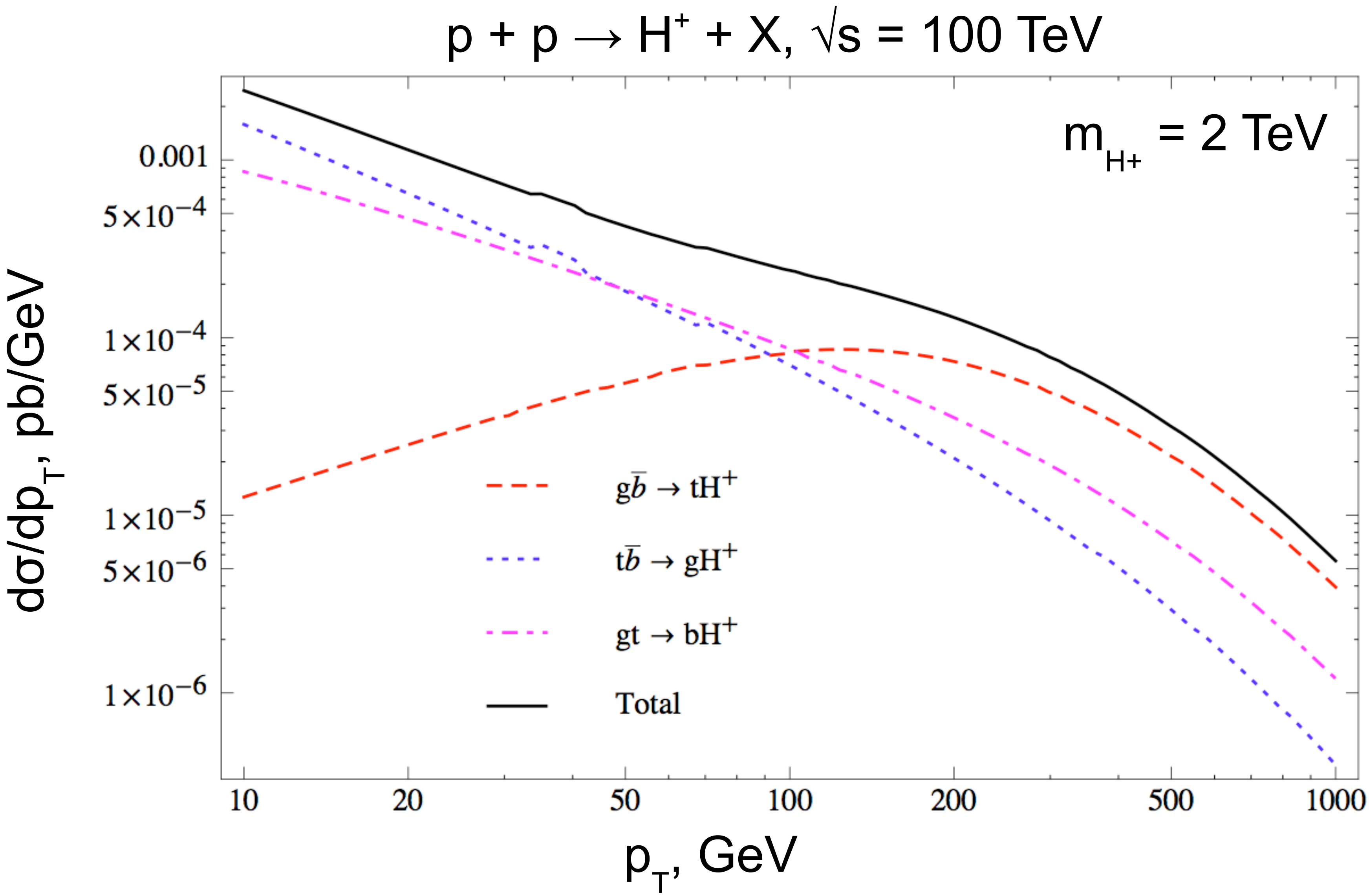
Charged Higgs production

- For production of charged Higgs plus X, turnover is roughly at $p_T \sim m_X$; this is more important than before!



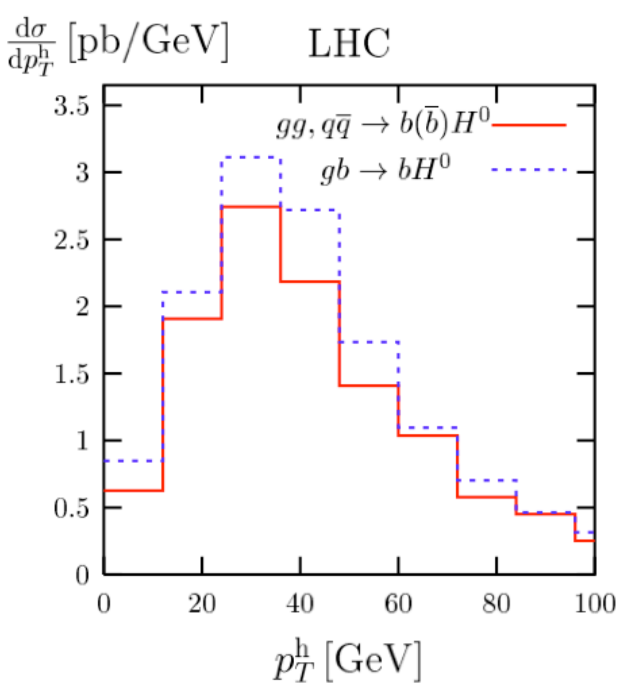
Charged Higgs production

- Mass effects at low p_T only included to LO in this calculation, using the S-ACOT (FONLL-A) scheme



Charged Higgs production

- For bottom quarks at the LHC, “low p_T” roughly corresponds to transverse momentum below the bottom mass, so this issue isn't as crucial
- Nevertheless, similar analogous studies suggest that we can do much better in predicting the charged Higgs p_T distribution in the 5FNS by going to NLO



p_T distribution for Higgs production in association with at least one b quark
NLO 4FNS vs. 5FNS

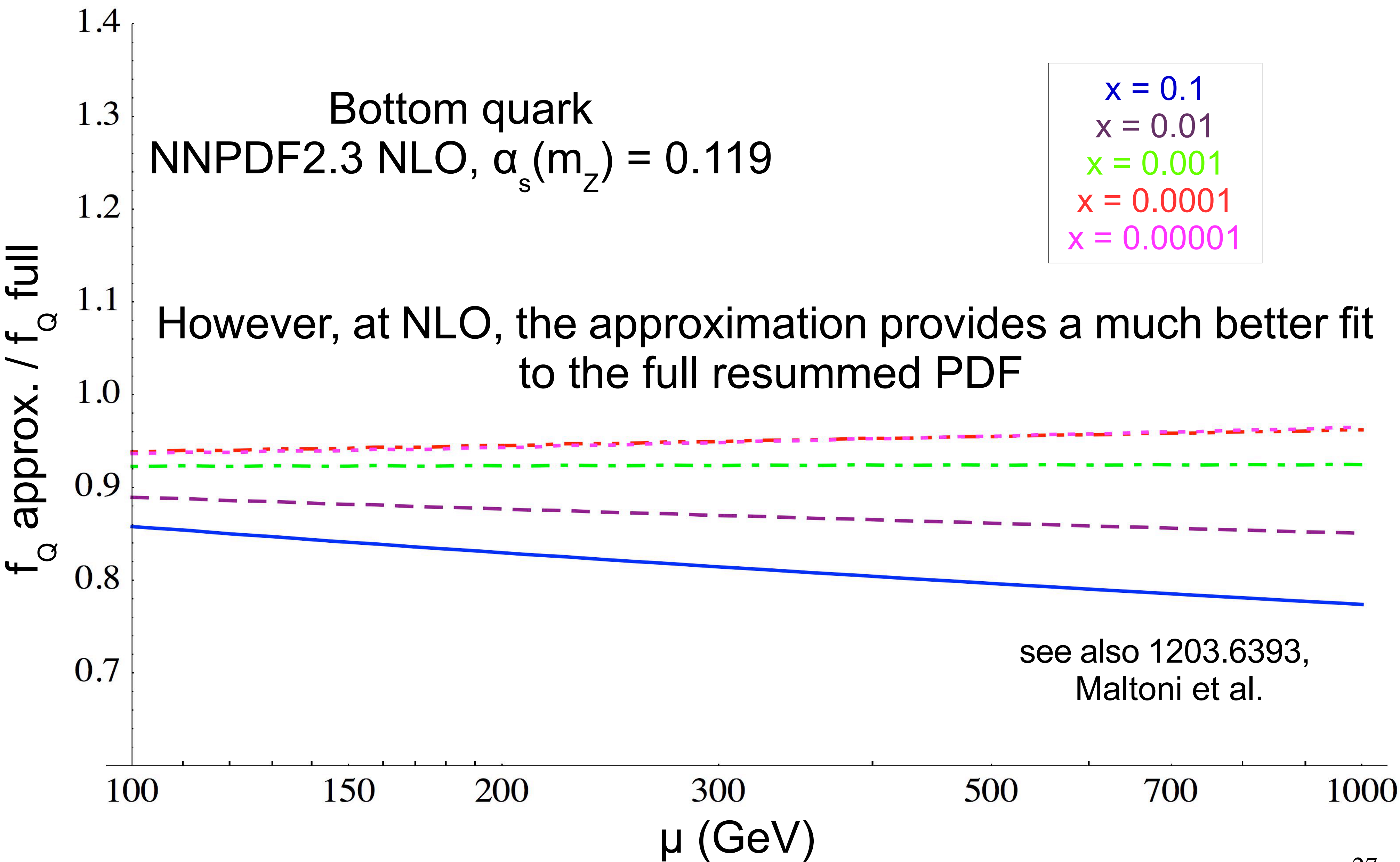
Dawson et al., hep-ph/0508293

Summary

- Because of α_s running and the heavy top mass, the gain from using a top PDF at a future pp collider is less than that from using a bottom PDF at the LHC
- At very high scales, effect of resummed logs contained in top PDF can change calculated cross sections by a factor of a few, which would seemingly translate into only slight changes in search reach
- However, kinematic distributions such as the p_T spectrum need more care, with effects that are *more* important for the top quark than for the bottom quark

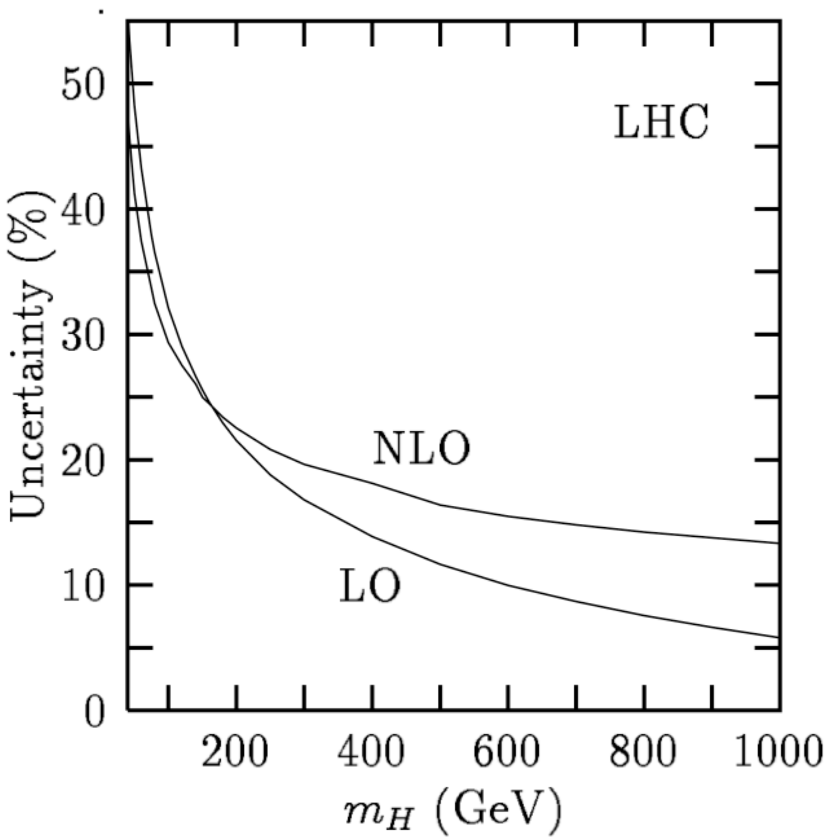
Backup

Heavy quark PDFs



Heavy quark PDFs

- So, for inclusive Higgs production in association with bottom quarks, the 4- and 5-flavor number schemes should give similar predictions at NLO for the LHC
- Scale uncertainties are sizable at NLO, unfortunately....

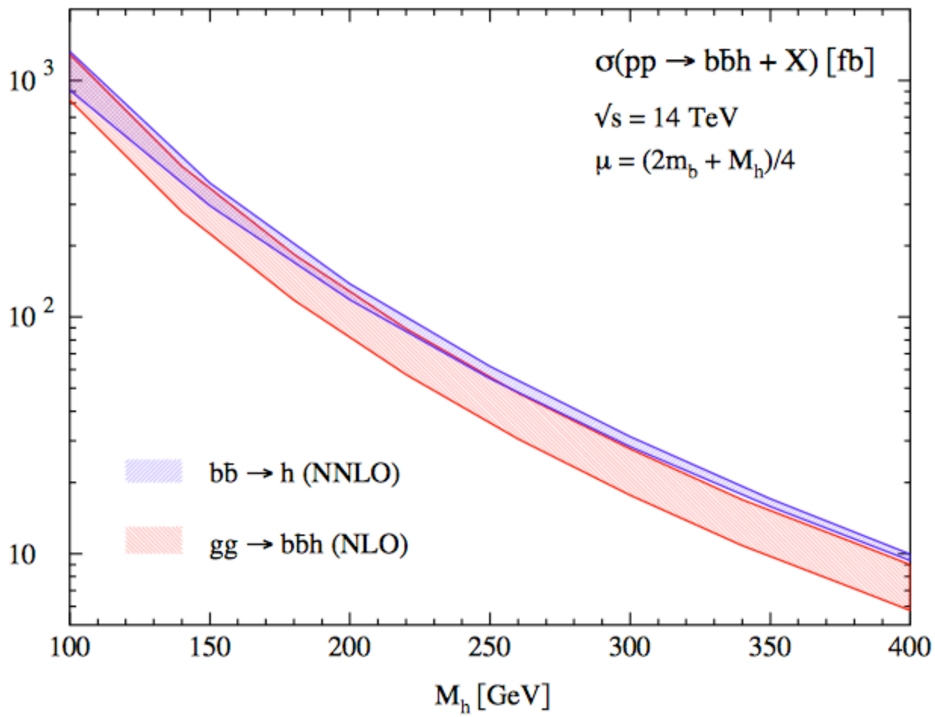


Scale uncertainty of NLO inclusive Higgs production in association with bottom quarks, calculated in 5-flavor number scheme

Dicus et al., hep-ph/9811492

Heavy quark PDFs

- After going to NNLO, different schemes agree quite well, with smaller scale uncertainties



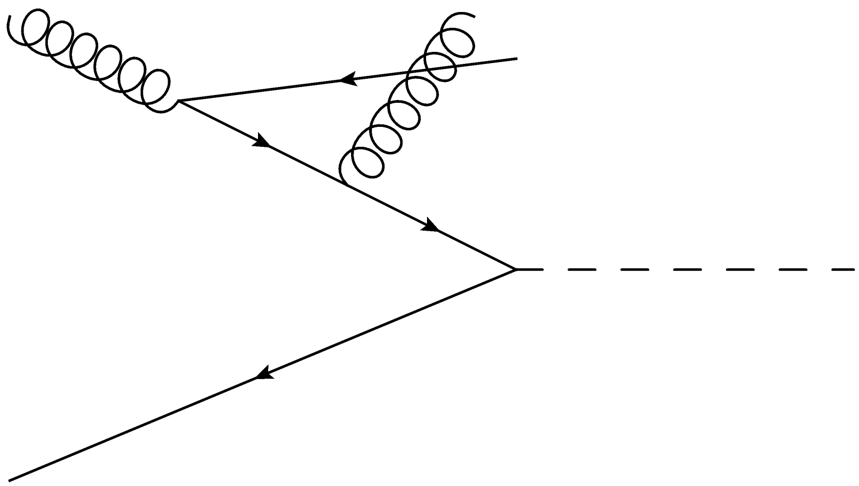
Inclusive Higgs production in association with bottom quarks, 4FNS vs. 5FNS

Campbell et al., hep-ph/0405302

- Much more has been said about the role of heavy quark PDFs in b-initiated Higgs processes at the LHC

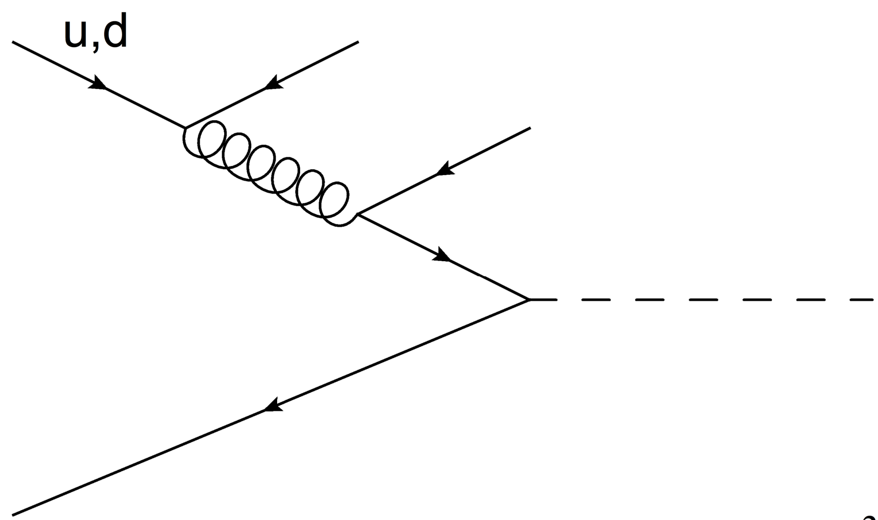
Charged Higgs production

- The cross section is now complete up to terms of order α_s^2 (log m_H / m_t) and higher
- Full NLL requires a few more components
  - NLO PDFs rather than LO PDFs



out of order splittings

splittings from light quarks



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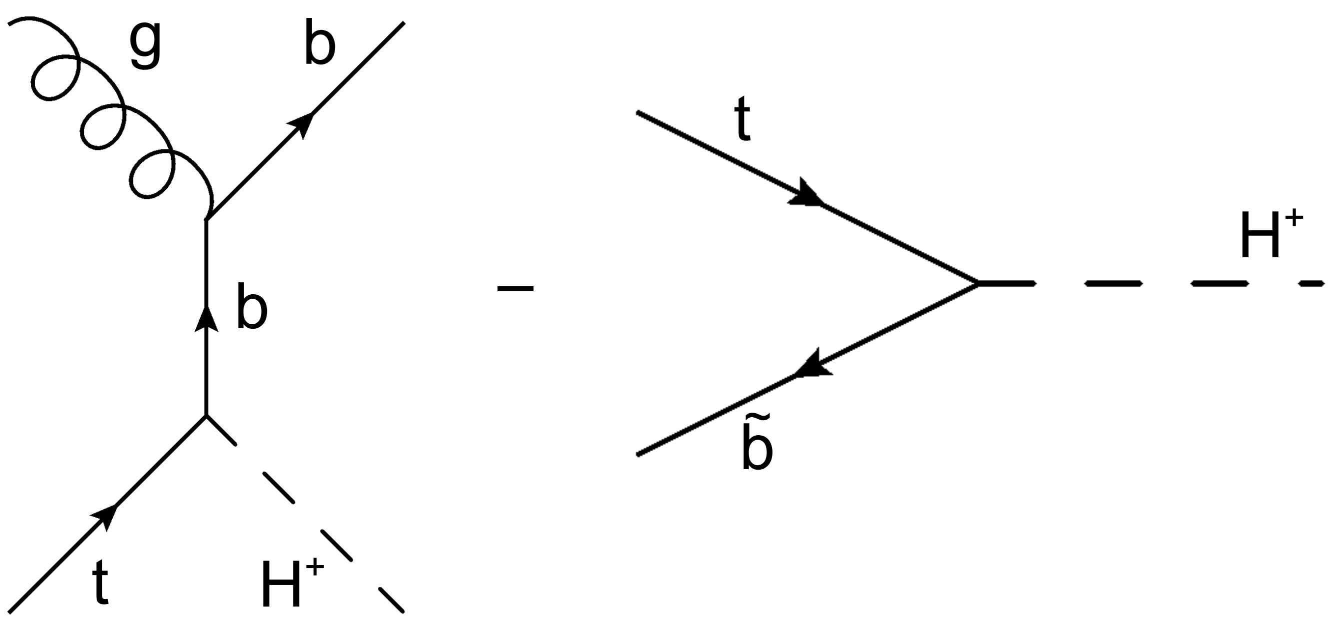

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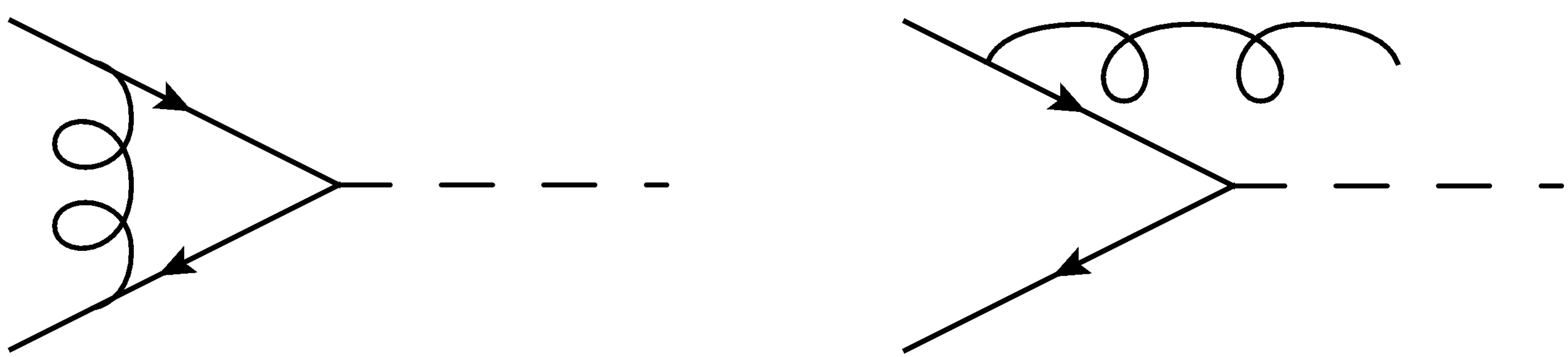

Mass effects

- Point of using a heavy quark PDF is to make predictions at scales >> the heavy quark mass
- At scales ~ the quark mass, finite mass effects enter
- S-ACOT: take heavy quark to be massless
- FONLL-A: LO massive quark function at low Q, NLO massless function at high Q (used by NNPDF2.3 NLO) *equivalent to S-ACOT*
- FONLL-B: NLO massive quark function at low Q, NLO massless function at high Q
- FONLL-C: NLO massive quark function at low Q, NNLO massless function at high Q